wave function

1. Express $8\cos x° - 6\sin x°$ in the form $k\cos(x° + a°)$ where $k > 0$ and $0 < a < 360$. **4**

2. (*a*) Write $\sin(x) - \cos(x)$ in the form $k\sin(x - a)$ stating the values of k and a where $k > 0$ and $0 \le a \le 2\pi$ **4**

    (*b*) Sketch the graph of $y = \sin(x) - \cos(x)$ for $0 \le x \le 2\pi$, showing clearly the graph's maximum and minimum values and where it cuts the x-axis and the y-axis. **3**

    3. (*a*) $12\cos x° - 5\sin x°$ can be expressed in the form $k\cos(x + a)°$, where $k > 0$ and $0 \le a < 360$.

    Calculate the values of k and a. **4**

    (*b*) (i) Hence state the maximum and minimum values of $12\cos x° - 5\sin x°$.

    (ii) Determine the values of x, in the interval $0 \le x < 360$, at which these maximum and minimum values occur. **3**

4. (*a*) Express $\sin x° - 3\cos x°$ in the form $k\sin(x - a)°$ where $k > 0$ and $0 \le a < 360$. Find the values of k and a. **4**

    (*b*) Find the maximum value of $5 + \sin x° - 3\cos x°$ and state a value of x for which this maximum occurs. **2**

5. Solve the simultaneous equations

$$k\sin x° = 5$$
$$k\cos x° = 2,$$

    where $k \ge 0$ and $0 \le x \le 360$. **4**

6. Express $2\sin x° - 5\cos x°$ in the form $k\sin(x - \alpha)°$, $0 \le \alpha < 360$ and $k > 0$. **4**

7. The formula $d = 200 + 80(\cos 30t° + \sqrt{3}\sin 30t°)$ gives an approximation to the depth of water, d, measured in centimetres, in a harbour t hours after midnight.

    (a) Express $f(t) = \cos 30t° + \sqrt{3}\sin 30t°$ in the form $k\cos(30t - \alpha)°$ and state the values of k and α, where $0 \le \alpha \le 360$. **(4)**

    (b) (i) Use your result from part (a) to help you sketch the graph of $f(t)$ for $0 \le t \le 12$.

    (ii) Hence, on a separate diagram, sketch the graph of d for $0 \le d \le 12$. **(6)**

    (c) What is the "low-water" time at the harbour during the time interval shown on your graph? **(1)**

    (d) If the local fishing fleet needs at least 1·5 metres depth of water to enter the harbour without risk of running aground, between what hours must it avoid entering the harbour during the time interval shown on your graph? **(2)**

8.

    (a) Show that $2\cos(x° + 30°) - \sin x°$ can be written as $\sqrt{3}\cos x° - 2\sin x°$. **3**

    (b) Express $\sqrt{3}\cos x° - 2\sin x°$ in the form $k\cos(x° + \alpha°)$ where $k > 0$ and $0 \le \alpha \le 360$ and find the values of k and α. **4**

    (c) Hence, or otherwise, solve the equation $2\cos(x° + 30°) = \sin x° + 1$, $0 \le x \le 360$. **3**

9. The displacement, d units, of a wave after t seconds, is given by the formula $d = \cos 20t° + \sqrt{3}\sin 20t°$.

    (a) Express d in the form $k\cos(20t° - \alpha°)$, where $k > 0$ and $0 \le \alpha \le 360$. **4**

    (b) Sketch the graph of d for $0 \le t \le 18$. **4**

    (c) Find, correct to one decimal place, the values of t, $0 \le t \le 18$, for which the displacement is 1·5 units. **3**

10. (a) Express $3\sin x° - \cos x°$ in the form $k\sin(x-\alpha)°$, where
$k > 0$ and $0 \le \alpha \le 90$. **(4)**

(b) Hence find algebraically the values of x between 0 and 180 for which
$3\sin x° - \cos x° = \sqrt{5}$. **(4)**

(c) Find the range of values of x between 0 and 180 for which
$3\sin x° - \cos x° \le \sqrt{5}$. **(2)**

11. $f(x) = 2\cos x° + 3\sin x°$.

(a) Express $f(x)$ in the form $k\cos(x-\alpha)°$ where $k > 0$ and $0 \le \alpha < 360$. **(4)**

(b) Hence solve algebraically $f(x) = 0.5$ for $0 \le x < 360$. **(3)**

12. The function f is defined by $f(x) = 2\cos x° - 3\sin x°$.

(a) Show that $f(x)$ can be expressed in the form $f(x) = k\cos(x+\alpha)°$ where
$k > 0$ and $0 \le \alpha < 360$, and determine the values of k and α. **(4)**

(b) Hence find the maximum and minimum values of $f(x)$ and the values of x
at which they occur, where x lies in the interval $0 \le x < 360$. **(4)**

(c) Write down the minimum value of $(f(x))^2$. **(1)**

[END OF QUESTIONS]